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An ECG Segmentation Method Based on GMM and Clusterwise Regression

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Abstract
The electrocardiogram (ECG) segmentation needs to separate different waves from an ECG and cluster the waves simultaneously. Clusterwise regression is a useful approach that can segment and cluster the data simultaneously. In this paper, we apply the clusterwise regression method to segment the ECG. By modeling the ECG signal wave by the Gaussian mixture model (GMM) and introducing a weight function, we propose a minimization model that consists of the weighted sum of the negative log-likelihood and the total variation (TV) of the weight function. The TV of the weight function enforces the temporal consistency. A supervised algorithm is designed to solve the proposed model. Experimental results show the efficiency of the proposed method for the ECG segmentation.

Keywords Electrocardiogram (ECG) · Segmentation · Fiducial point extraction · Gaussian mixture model (GMM) · Clusterwise regression

Mathematics Subject Classification 92C55 · 62H30 · 62R07

1 Introduction

Electrocardiogram (ECG) shows the electrical changes of the human heart. The ECG signal morphologies provide essential people's cardiac information which is important for early heart disease detection, so that people can prevent themselves from being threatened by cardiac diseases. The ECG is a quasi-periodic biomedical signal of all the cardiac cycles. One cardiac cycle represents the information of a single periodic heartbeat. A typical cardiac cycle in an ECG signal contains the P wave, QRS complex, T wave, and PQ, ST intervals. Figure 1 is an ECG signal illustration of a complete cardiac cycle. The shapes, amplitudes of the waves, and the time intervals between any wave in the current cardiac cycle and waves

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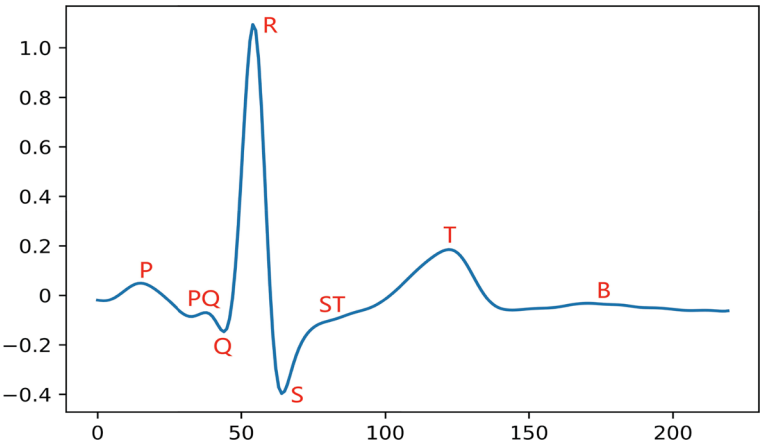

Fig. 1 An ECG signal of a complete cardiac cycle

in the next cycle can indicate whether the heart is healthy. Clinical detection for pathological beats and heart diseases is based on such intervals and the characteristics of different waves. Therefore, the proper segmentation of ECG signals is very important for the accurate heart disease diagnosis. The ECG segmentation aims to separate the ECG into different wave subsegments, determine the wave type clusters of these subsegments, and detect the fiducial points of each wave that include the onset, offset, and peak of P, QRS, and T waves.

Many approaches have been proposed for the ECG segmentation to detect the QRS complex, ST-Segment, R-peak, and other fiducial points. For instance, Di Marco and Chiari [11] proposed a wavelet-based ECG delineation algorithm, which can be used for the online QRS detection and P-QRS-T waves delineation of a single lead ECG signal; Fujita et al. [3] explored the performance of the wavelet-based ECG analysis method for the ST-segment detection; Umer et al. [20] proposed an ECG feature extraction and pattern recognition method using a novel windowing algorithm; in Ref. [2], Curtin et al. proposed an automated approach for the QRS complex detection and QRS duration measurement, which can effectively analyze multichannel ECG signals obtained from cardiac resynchronization therapy patients; in Ref. [22], Xu et al. proposed a rule-based method for morphological classification of ST segments in ECG signals that can identify ST segments with the normal morphology type and five abnormal morphology sub-types; in Ref. [8], Hu and Bao proposed an approach to the QRS complex detection based on multiscale mathematical morphology; in Ref. [5], Goovaerts et al. proposed a machine-learning approach for the detection and quantification of the QRS fragmentation; in Ref. [6], Hadjem et al. proposed an ST-segment and T wave anomaly prediction method in the ECG data using RUSBoost; in Ref. [21], Xiao et al. monitored significant ST changes through deep learning; in Ref. [1], Akhbari proposed an ECG segmentation and fiducial point extraction method using the multihidden Markov model. Most of these methods can detect only one wave feature, or first detect the QRS complexes and then search the P wave and T wave according to the location of the QRS.

The ECG signal segmentation can be fulfilled by some clustering methods [15, 19]. Clusterwise regression is a clustering method, that assumes that the data in the same cluster follows the same regression model, and classifies data by minimizing the sum of regression errors [16, 17]. The Gaussian mixture model (GMM) is a statistical model that can be used to detect the ECG anomalies and model the ECG signals. For example, in Ref. [12], Martis et

al. proposed a two-stage mechanism for the registration and classification of an ECG using the GMM; in Ref. [18], based on the GMM, Terzı and Arikan proposed an unsupervised learning technique to detect the myocardial ischemia; in Ref. [13], McSharry et al. suggested the use of the GMM to generate synthetic ECG signals; in Ref. [14], Mneimneh and Povinelli proposed an RPS/GMM approach toward the localization of the myocardial infarction, where the GMM is used to model the embedded ECG signal. Based on the clusterwise regression and GMMs, we focus on representing each type of the wave by the GMM to cluster the ECG signal and obtain ECG signal segmentation results.

In order to obtain good ECG segmentation results, the temporal consistency of the ECG signal is considered. The temporal consistency assumes that successive signal data belong to the same type of the wave and it encourages the adjacent points to be grouped into the same cluster [7]. In this paper, we propose a weight function representing the possibility that each data falls into the clusters. Using the clusterwise regression method, the GMM for each type of the wave and the weight function, the weighted sum of the negative log-likelihood is minimized to segment and cluster the ECG signals. The total variation (TV) of the weight function is applied in the minimization model to fulfill the temporal consistency. A supervised algorithm is designed to solve the proposed model. In the training step, a rough segmentation is performed to define the training data for each GMM. Then, the expectation-maximization (EM) algorithm is used to determine the parameters of each GMM separately. In the test step, the label of the current beat subsegment is estimated by calculating the weight function.

The rest of the paper is organized as follows. In Sect. 2, we propose a minimization model for the ECG segmentation. In Sect. 3, we develop a supervised algorithm to solve the proposed model. In Sect. 4, some experiments are performed to demonstrate the effectiveness of the proposed method. Finally, a conclusion is given in Sect. 5.

2 Methods

For recorded ECG signals $s_1, s_2, \cdots, s_\tau$, we define its first-order difference as $y_1 = s_1$, $y_t = s_t - s_{t-1}$, $t = 2, 3, \cdots, \tau$. Then, we get the two-dimensional ECG signal series $x_t = [s_t, y_t]'$ $(t = 1, 2, \cdots, \tau)$. We cluster $x_1, x_2, \cdots, x_\tau$, and get its segmentation results. The maximum or minimum value of each subsegment is associated with the location of its peak. From the probability distribution histograms and Gaussian kernel fitting of different waveforms and intervals of an example ECG signal shown in Fig. 2, we find that each type of wave and interval can be represented by a GMM.

That is, suppose that the signal can be divided into C types of waves, if x_t $(t = 1, 2, \cdots, \tau)$ belongs to the c-th cluster $(c \in \{1, 2, \cdots, C\})$, it can be assumed to follow the following Gaussian mixture distribution:

$$p\left(x_t|\mu^c, \Gamma^c\right) = \sum_{k=1}^{K^c} \alpha_k^c \mathcal{N}\left(x_t|\mu_k^c, \Gamma_k^c\right),$$

where K^c is the number of the components of the Gaussian mixture, α_k^c is the mixture weight with the constraint that the weights sum to unity, $\mathcal{N}(x_t|\mu_k^c, \Gamma_k^c)$ represents a multivariate Gaussian probability density function, and $\mu^c = (\mu_1^c, \mu_2^c, \cdots, \mu_{K^c}^c)$ and $\Gamma^c = (\Gamma_1^c, \Gamma_2^c, \cdots, \Gamma_{K^c}^c)$ are the mean vector and covariance matrix of the multivariate Gaussian variables, respectively. According to the wave types of the ECG signal, we assume the number of wave types or clusters is $C = 6$. For $t = 1, 2, \cdots, \tau$ and $c = 1, 2, \cdots, C$, by utilizing the negative

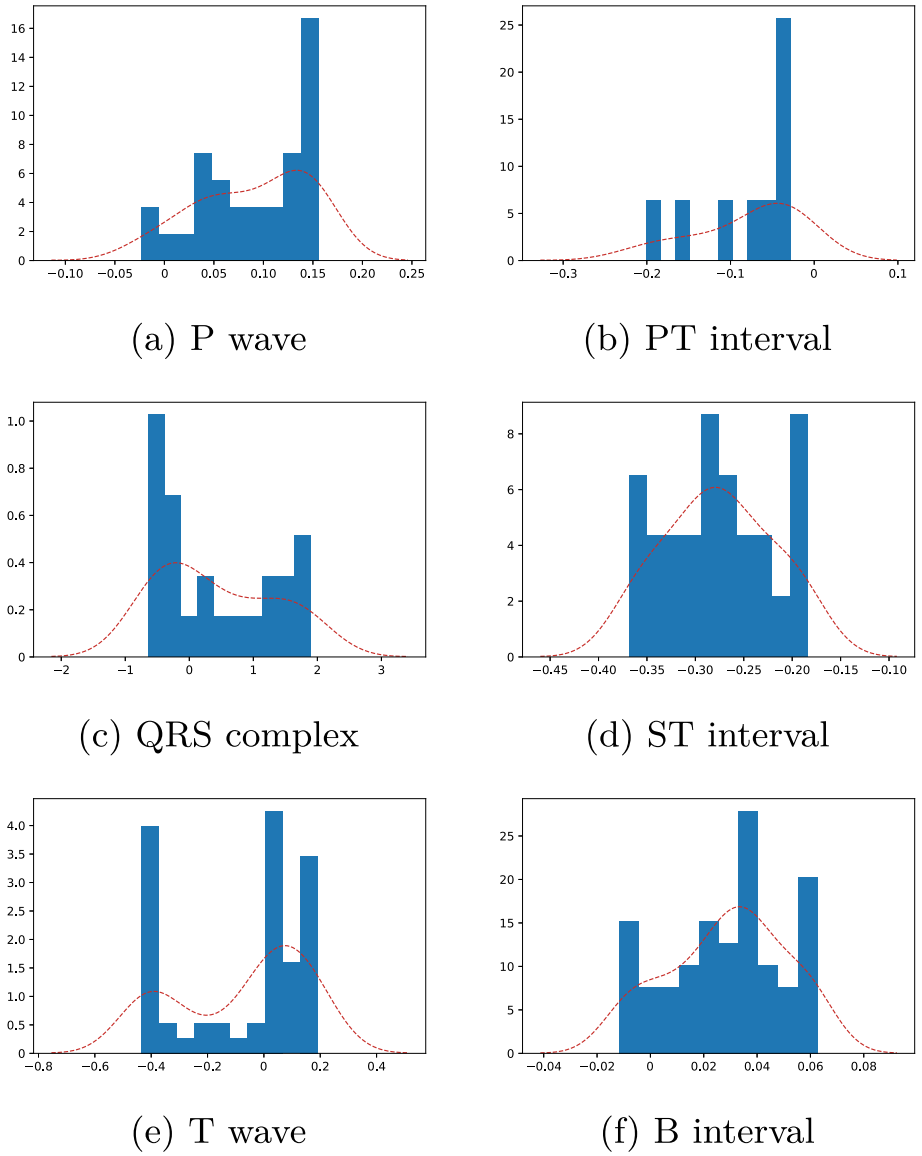

Fig. 2 Probability distribution histograms and Gaussian kernel fitting for different waves and intervals of an example ECG signal

log-likelihood function, we define the similarity measure between x_t and the c-th cluster as

$$d(x_t, \mu^c, \Gamma^c) = -\log(p(x_t|\mu^c, \Gamma^c)).$$

For x_t $(t = 1, 2, \cdots, \tau)$, we introduce the function $f_c(x_t)$ which means the probability that x_t belongs to the c-th cluster. Since x_t can only fall into one cluster and if we know such

cluster is the $\hat{c}$-th cluster, we can define

$$f_c(x_t) = \begin{cases} 1, \ c = \hat{r}; \\ 0, \ c \in \{1, 2, \cdots, C\}, c \neq \hat{c}. \end{cases}$$

It holds $\sum_{c=1}^{C} f_c(x_t) = 1$ for any $t = 1, 2, \cdots, \tau$. Therefore, we can take $f_c(x_t)$ as the weight function. Denote $f_c = (f_c(x_1), f_c(x_2), \cdots, f_c(x_\tau))'$, $F = (f_1, f_2, \cdots, f_C)'$, and $\mathbf{1}$ is a column vector of the length τ whose elements are all 1. We construct the following constrained minimization model to solve the problem by combining the weighted sum of negative log-likelihood:

$$\begin{aligned} \min_{F,\mu,\Gamma} L(F, \mu, \Gamma) &= \sum_{c=1}^{C} \sum_{t=1}^{\tau} d\left(x_t, \mu^c, \Gamma^c\right) f_c(x_t), \\ \text{s.t} \sum_{c=1}^{C} f_c &= \mathbf{1}, \ f_c \in [0, 1]^\tau, \ c = 1, 2, \cdots, C, \end{aligned} \tag{1}$$

where $\mu = (\mu^1, \mu^2, \cdots, \mu^C)$ and $\Gamma = (\Gamma^1, \Gamma^2, \cdots, \Gamma^C)$ are the parameters of the Gaussian mixture models. We define the TV of the weight function matrix F in the temporal direction as

$$\|F\|_{\mathrm{TV}} = \sum_{t=1}^{\tau-1} \sum_{c=1}^{C} \left| f_c(x_{t+1}) - f_c(x_t) \right|.$$

It is obvious that $\|F\|_{\mathrm{TV}}$ can be used to judge the temporal consistency of the ECG signal. Smaller values of $\|F\|_{\mathrm{TV}}$ force adjacent data points to belong to the same cluster. Denote

$$d_{(\mu^c, \Gamma^c)} = \left(d\left(x_1, \mu^c, \Gamma^c\right), d\left(x_2, \mu^c, \Gamma^c\right), \cdots, d\left(x_\tau, \mu^c, \Gamma^c\right)\right)',$$

we expect to get the subsegments with the temporal consistency constraint. Therefore, the TV of the weight function matrix is introduced into (1) as the regularization term, and the segmentation results are obtained by solving the following minimization problem:

$$\begin{aligned} \min_{F,\mu,\Gamma} \sum_{c=1}^{C} d'_{(\mu^c, \Gamma^c)} f_c + \beta \|F\|_{\mathrm{TV}}, \\ \text{s.t.} \sum_{c=1}^{C} f_c = \mathbf{1}, \ f_c \geqslant 0, \ c = 1, 2, \cdots, C, \end{aligned} \tag{2}$$

where $\beta > 0$ is a positive constant.

3 Algorithm

In this section, we design a supervised algorithm to minimize the model (2). We first use 50% of the data as the training set to train each GMM model. Then, using the remaining 50% of the data as the test set, we introduce the trained GMM models into the model (2) and obtain the clustering results of the test set by solving the function F in the model (2). In the training process, the number of Gaussian mixture components K^c ($c = 1, 2, \cdots, C$) is estimated by the AIC or BIC criterion.

3.1 Training

In the algorithm, each GMM corresponding to a certain type of wave is trained separately. For example, when we train the GMM of the P wave, we take all the P waves of the 50% ECG signal to be the training data, and then we complete the training of the GMM corresponding to the P wave. For the other GMMs, we repeat this procedure and finally get the GMMs for all the wave types. In each training, the expectation-maximization (EM) algorithm is used to fit each GMM. Let X^c_{train} represent the training set composed of all c-th waves of 50% of the training ECG signal, $x_t \in X^c_{\text{train}}$. Given initial $(\alpha^c_k)^{(0)}$, $(\mu^c_k)^{(0)}$, $(\Gamma^c_k)^{(0)}$, for $i = 1, 2, \cdots$, we fit the c-th ($c = 1, 2, \cdots, C$) GMM model by the following EM process.

- Calculate $\omega^c_{t,k}$:

$$\left(\omega^c_{t,k}\right)^{(i)} = \frac{\left(\alpha^c_k\right)^{(i-1)} \mathcal{N}\left(x_t|(\mu^c_k)^{(i-1)}, (\Gamma^c_k)^{(i-1)}\right)}{\sum_{k=1}^{K^c}\left(\alpha^c_k\right)^{(i-1)} \mathcal{N}\left(x_t|(\mu^c_k)^{(i-1)}, (\Gamma^c_k)^{(i-1)}\right)},$$

  where $(\omega^c_{t,k})^{(i)} = p(z = k|x_t, (\mu^c_k)^{(i-1)}, (\Gamma^c_k)^{(i-1)})$ is the probability that a certain data point x_t belongs to the k-th Gaussian model after calculating the parameters of the GMM.
- Update the parameter $(\alpha^c_k)^{(i)}$:

$$\left(\alpha^c_k\right)^{(i)} = \frac{\sum_{t\in X^c_{\text{train}}}\left(\alpha^c_k\right)^{(i-1)}}{\left|X^c_{\text{train}}\right|},$$

  where $|X^c_{\text{train}}|$ is the number of elements in the set X^P_{train}.
- Update the parameter $(\mu^c_k)^{(i)}$:

$$\left(\mu^c_k\right)^{(i)} = \frac{\sum_{t\in X^c_{\text{train}}}\left(\omega^c_{t,k}\right)^{(i-1)} x_t}{\sum_{t\in X^c_{\text{train}}}\left(\omega^c_{t,k}\right)^{(i-1)}}.$$

- Update the parameter $(\Gamma^c_k)^{(i)}$:

$$\left(\Gamma^c_k\right)^{(i)} = \frac{\sum_{t\in X^c_{\text{train}}}\left(\omega^c_{t,k}\right)^{(i-1)}\left[x_t - (\mu^c_k)^{(i-1)}\right]'\left[x_t - (\mu^c_k)^{(i-1)}\right]}{\sum_{t\in X^c_{\text{train}}}(\omega^c_{t,k})^{(i-1)}}.$$

3.2 Test

After training all GMMs, we obtain the parameters of all GMMs, denoted as $\hat{\mu} = (\hat{\mu}^1, \hat{\mu}^2, \cdots, \hat{\mu}^C)$, $\hat{\Gamma} = (\hat{\Gamma}^1, \hat{\Gamma}^2, \cdots, \hat{\Gamma}^C)$, then for the data x_t in the test set ($x_t \in X_{\text{test}}$), we can calculate the log-likelihood of each GMM as

$$d(x_t, \hat{\mu}^c, \hat{\Gamma}^c) = -\log(p(x_t|\hat{\mu}^c, \hat{\Gamma}^c)),\ c = 1, \cdots, C.$$

Afterwards, for each $x_t \in X_{\text{test}}$, we solve the following problem to obtain the weight function F:

$$\min_F \sum_{c=1}^{C} d'_{\left(\hat{\mu}^c, \hat{\Gamma}^c\right)} f_c + \beta\|F\|_{\text{TV}}, \tag{3}$$

$$\text{s.t.} \sum_{c=1}^{C} f_c = \mathbf{1},\ f_c \geqslant 0,\ c = 1, 2, \cdots, C.$$

Let D denote the gradient matrix. Then we can rewrite

$$\|F\|_{\text{TV}} = \sum_{c=1}^{C} \|Df_c\|_1.$$

By introducing the auxiliary variable $G = (g_1, g_2, \cdots, g_C)'$, we reformulate (3) as

$$\begin{aligned} &\min_{F,G} \sum_{c=1}^{C} \left(d'_{\left(\hat{\mu}^c, \hat{\Gamma}^c\right)} f_c + \beta \|g_c\|_1 \right), \\ &\text{s.t.} \sum_{c=1}^{C} f_c = \mathbf{1},\ g_c = Df_c,\ f_c \geqslant 0,\ c = 1, 2, \cdots, C. \end{aligned} \tag{4}$$

The augmented Lagrangian functional of (4) is

$$\begin{aligned} \min_{\substack{f_c \geqslant 0, g_c, \\ c=1,2,\cdots,C}} & \sum_{c=1}^{C} \left(d'_{\left(\hat{\mu}^c, \hat{\Gamma}^c\right)} f_c + \beta \|g_c\|_1 + z'_c(g_c - Df_c) + \frac{\gamma}{2} \|g_c - Df_c\|_2^2 \right) \\ & + y' \left(\sum_{c=1}^{C} f_c - 1 \right) + \frac{\lambda}{2} \left\| \sum_{c=1}^{C} f_c - 1 \right\|_2^2, \end{aligned}$$

where $z_1, z_2, \cdots, z_C$, and y are Lagrange multipliers, and $\gamma, \lambda > 0$ are penalty parameters. For $i = 1, 2, \cdots$, $c = 1, 2, \cdots, C$, the process of the alternating direction method of multipliers (ADMM) is given as

$$\begin{aligned} f_c^{(i)} = &\arg \min_{f_c \geqslant 0} d'_{\left(\hat{\mu}^c, \hat{\Gamma}^c\right)} f_c - \left(z_c^{(i-1)} \right)' Df_c + \frac{\gamma}{2} \left\| g_c^{(i-1)} - Df_c \right\|_2^2 \\ & + \left(y^{(i-1)} \right)' f_c + \frac{\lambda}{2} \left\| f_c + \sum_{k=1}^{c-1} f_k^{(i)} + \sum_{k=c+1}^{C} f_k^{(i-1)} - 1 \right\|_2^2, \end{aligned} \tag{5}$$

$$g_c^{(i)} = \arg \min_{g_c} \beta \|g_c\|_1 + \left(z_c^{(i-1)} \right)' \left(g_c - Df_c^{(i)} \right) + \frac{\gamma}{2} \left\| g_c - Df_c^{(i)} \right\|_2^2, \tag{6}$$

$$z_c^{(i)} = z_c^{(i-1)} + \gamma \left(g_c^{(i)} - Df_c^{(i)} \right), \tag{7}$$

$$y^{(i)} = y^{(i-1)} + \lambda \left(\sum_{c=1}^{C} f_c^{(i)} - 1 \right). \tag{8}$$

Let $UU = \gamma D'D + \lambda I$. Then the subproblem (5) can be rewritten as

$$f_c^{(i)} = \arg \min_{f_c \geqslant 0} \frac{1}{2} \left\| Uf_c - U^{-1}\xi \right\|_2^2 + \text{con}_1 \tag{9}$$

with

$$\xi = -d_{\left(\hat{\mu}^c, \hat{\Gamma}^c\right)} + D' z_c^{(i-1)} + \gamma D' g_c^{(i-1)} - y^{(i-1)} - \lambda \left(\sum_{k=1}^{c-1} f_k^{(i)} + \sum_{k=c+1}^{C} f_k^{(i-1)} - 1 \right), \tag{10}$$

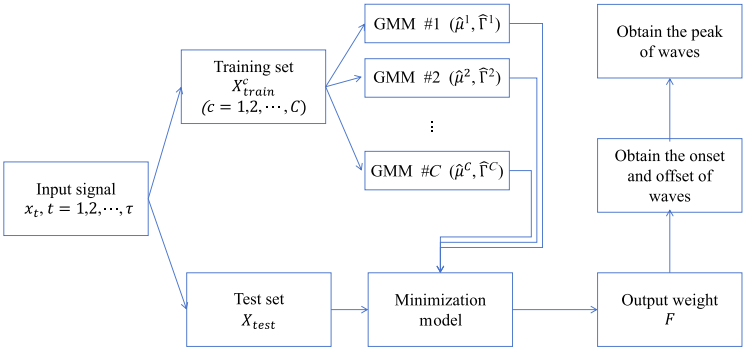

Fig. 3 Block diagram of the proposed supervised algorithm for a fiducial points detection

where con_1 is a constant independent of f_c, and U is a symmetric matrix. The constrained problem (9) with the constraint (10) is convex, and according to KKT conditions, we obtain the solution of f_c as

$$f_c^{(i)} = \max\left\{0, (\gamma D'D + \lambda I)^{-1}\xi\right\}.$$

The subproblem (6) can be written as

$$g_c^{(i)} = \arg\min \frac{\beta}{\gamma}\|g_c\|_1 + \frac{1}{2}\left\|g_c - (Df_c^{(i)} - \frac{1}{\gamma}z_c^{(i-1)})\right\|_2^2 + \text{con}_2,$$

where con_2 is a constant independent of g_c. Let $[g_c]_j$ represent the j-th component of $[g_c]$. Then for $j = 1, 2, \cdots, N$, we have

$$[g_c^{(i)}]_j = \begin{cases} \left[Df_c^{(i)} - \frac{1}{\gamma}z_c^{(i-1)}\right]_j - \frac{\beta}{\gamma}, & \left[Df_c^{(i)} - \frac{1}{\gamma}z_c^{(i-1)}\right]_j - \frac{\beta}{\gamma} > 0, \\ \left[Df_c^{(i)} - \frac{1}{\gamma}z_c^{(i-1)}\right]_j + \frac{\beta}{\gamma}, & \left[Df_c^{(i)} - \frac{1}{\gamma}z_c^{(i-1)}\right]_j + \frac{\beta}{\gamma} < 0, \\ 0, & \text{others.} \end{cases}$$

Figure 3 shows the block diagram of the proposed supervised algorithm for determining the peak, onset, and offset of the ECG waves.

4 Experiments

In this section, we demonstrate the effectiveness of the proposed method through some ECG signal segmentation experiments using the ECG signal from the PhysioNet QT dataset. The PhysioNet QT dataset is available at the website https://www.physionet.org/physiobank/database/qtdb/. This QT dataset consists of ECG signals from 105 patients, where each patient's data is a 2-by-225 000 matrix, and each row of the data matrix corresponds to a lead ECG record for that patient. In the following experiments, we use the data recorded by the first lead. We separate the data into two parts with the equal size, then do training on the first part and testing on the second part. Meantime, do training on the second part and testing on the first part, and finally calculate the macro-F1 scores of the two tests. Since the annotations for onsets of the T wave are not available in the QT dataset, we do not calculate

Table 1 Macro-F1 scores of the ECG fiducial point extraction on the first experiment ("–" means not available)

Method	P_{on}	P_{peak}	P_{off}	QRS_{on}	R_{peak}	QRS_{off}	T_{peak}	T_{off}
Proposed	**0.98**	0.95	**0.98**	**1.00**	**1.00**	**1.00**	**0.98**	**0.98**
Peak	0.98	**0.98**	–	0.98	0.98	0.98	0.98	0.98
DWT	0.98	0.98	0.98	0.98	0.98	0.98	0.00	0.00
CWT	0.97	0.97	0.97	0.98	0.98	0.98	0.97	0.81

Table 2 Macro-F1 scores of the ECG fiducial point extraction on the second experiment ("–" means not available)

Method	P_{on}	P_{peak}	P_{off}	QRS_{on}	R_{peak}	QRS_{off}	T_{peak}	T_{off}
Proposed	0.97	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.97**
Peak	0.90	0.97	–	0.97	0.97	0.97	0.14	0.10
DWT	**0.98**	0.97	0.97	0.97	0.97	0.97	0.97	0.93
CWT	0.00	0.00	0.00	0.97	0.97	0.97	0.27	0.27

the estimation error for the onsets of the T wave. The hyperparameters in the algorithm are set to $\beta = 10,\ \lambda = 10,$ and $\gamma = 10$.

In the first experiment, we use the ECG signal "ecg3" from the QT dataset. The signal "ecg3" has 30 annotated beats and contains 6 363 sample points. We apply the proposed algorithm to this ECG signal, and the classical Peak-based method, the discrete wavelet transform (DWT) method, and the continuous wavelet transform (CWT) method are also applied to segment this ECG signal for comparisons. The code for these comparison methods is provided in Ref. [10]. The macro-F1 scores obtained by the different methods are shown in Table 1. For the proposed method, except for the macro-F1 scores of the P peak, the macro-F1 scores of other fiducial points are greater than or equal to the macro-F1 scores obtained by the Peak method, DWT, and CWT method. The macro-F1 scores of the T peak and T offset obtained by the DWT method are both 0, indicating that the DWT method can not accurately detect the location of the T wave in this experiment. For a more intuitive observation, we also show the fiducial points for the first 1 000 ECG signal data obtained by the proposed method in Fig. 4. From Fig. 4, we can also see that the proposed method can relatively accurately detect the fiducial points.

In the second experiment, we use the ECG signal "ecg7" from the QT dataset. This signal has 29 annotated beats and 9 575 sample points. The proposed algorithm, the Peak method, the DWT method, and the CWT method are also applied to segment this ECG signal. Table 2 shows the macro-F1 scores obtained by these four methods. As we can see from Table 2, the proposed method obtains relatively high macro-F1 scores, while the Peak method and the CWT method are unable to accurately detect the T waves and P waves, respectively. The fiducial points for the first 1 000 ECG signal data obtained by the proposed method are shown in Fig. 5, which shows that the proposed method can relatively accurately detect the fiducial points.

In the third experiment, we use the ECG signal "ecg30" from the QT dataset, which has 30 annotated beats and 6 467 sample points. This ECG signal is also segmented by four methods: the proposed algorithm, the Peak method, the DWT method, and the CWT method. We show the macro-F1 scores obtained by these four methods in Table 3. From Table 3, the

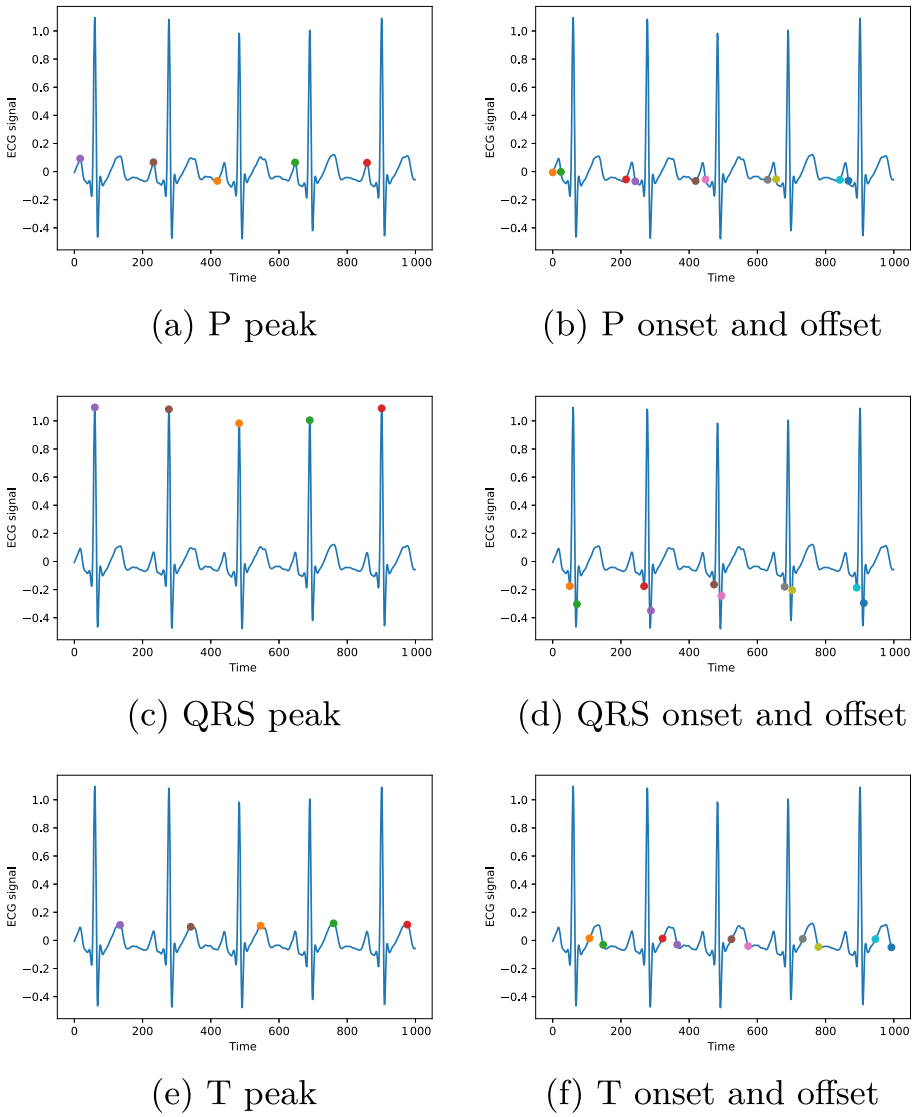

Fig. 4 The fiducial points obtained by the proposed method for the first 1 000 data points in "ecg3"

macro-F1 scores obtained by the proposed method are greater than or equal to the macro-F1 scores obtained by other methods. Figure 6 shows the fiducial points for the first 1 000 ECG signal data obtained by the proposed method, and it also shows that the proposed method can relatively accurately detect the fiducial points.

It can be seen from the above experiments that the proposed method can obtain relatively good detection results, while the Peak, DWT, and CWT methods may fail in the P wave or the T wave detection. Therefore, the proposed method outperforms the Peak, DWT, and CWT methods.

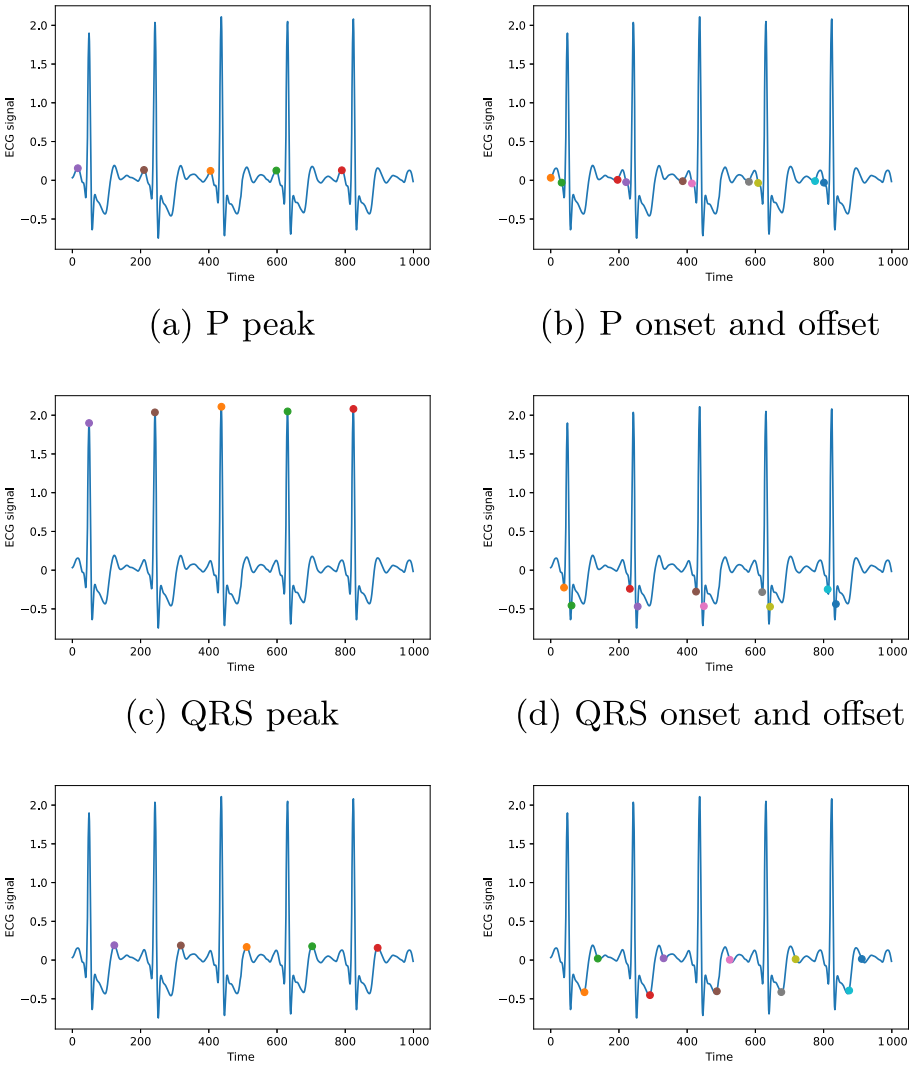

Fig. 5 The fiducial points obtained by the proposed method for the first 1 000 data points in "ecg7"

Table 3 Macro-F1 scores of the ECG fiducial point extraction on the third experiment ("–" means not available)

Method	P_{on}	P_{peak}	P_{off}	QRS_{on}	R_{peak}	QRS_{off}	T_{peak}	T_{off}
Proposed	**0.98**	**0.98**	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
Peak	0.98	0.98	–	0.98	0.98	0.98	0.98	0.03
DWT	0.98	0.98	0.95	0.98	0.98	0.95	0.95	0.07
CWT	0.00	0.00	0.00	0.98	0.98	0.98	0.97	0.70

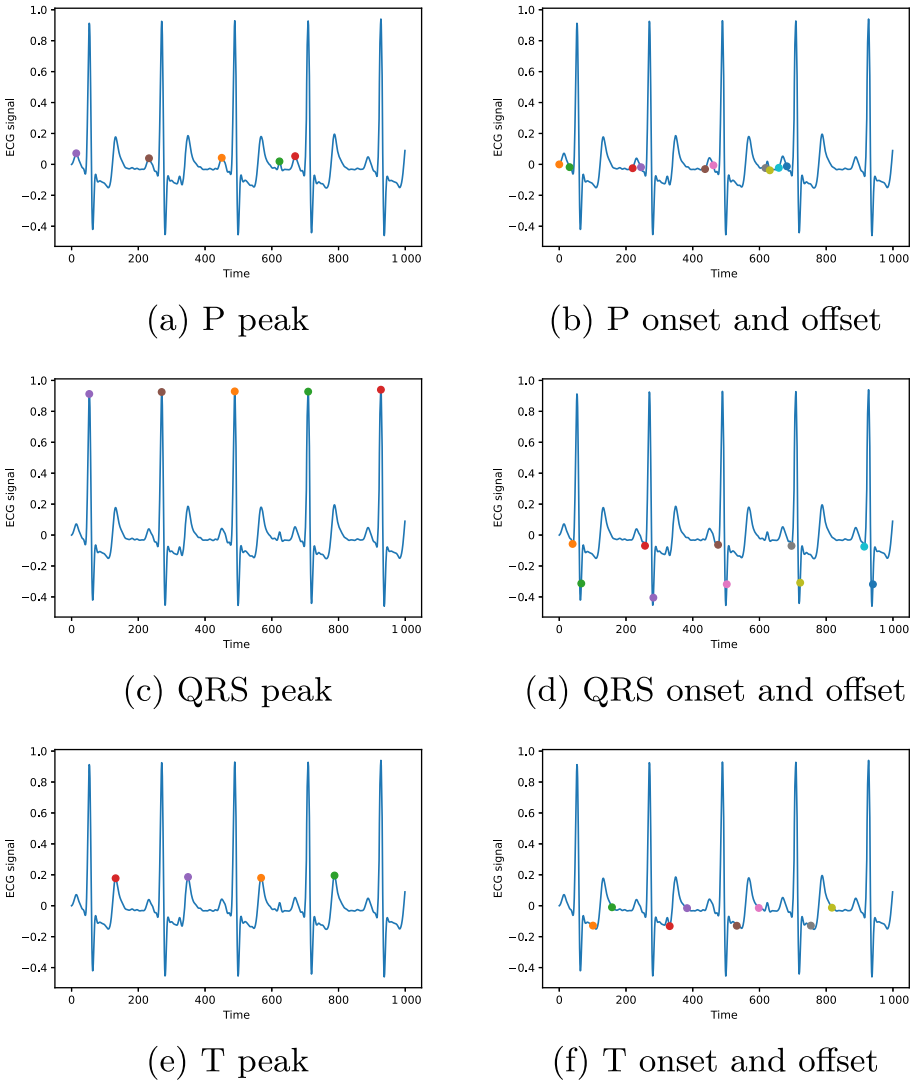

Fig. 6 The fiducial points obtained by the proposed method for the first 1 000 data points in "ecg30"

5 Summary

The clusterwise regression is a clustering technique that can simultaneously segment and cluster data. In this paper, based on the clusterwise regression method, GMMs, and the weight function, we propose a minimization model for the ECG segmentation and fiducial point extraction. In the proposed method, each wave of the ECG signal is modeled by a GMM. By introducing a weight function, we cluster the ECG sample points by minimizing the weighted sum of the negative log-likelihood of all GMMs. To ensure the temporal consistency of the data, the total variation regularization of the weight function is also introduced into the model. A supervised algorithm is used to minimize the proposed model. Experimental results show that the proposed method is efficient for finding the fiducial points of the ECG signal.

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Declarations

Conflict of Interest Jonghoon Kim was the Associate Editor of the Journal of Power Electronics while conducting this study. Editorial Board Member status has no bearing on editorial consideration.

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